Energy-Neutral System-Level Analysis and Optimization of 5G Wireless Networks

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Energy-neutral network: A self-sustained network, that is able to operate without requiring energy from the electrical supply.

Very ambitious goal, which requires two components:

- **Energy efficiency maximization**: optimize the system radio resources to make the most efficient use of the available energy.
- **Energy harvesting and transfer** to obtain new energy and redistribute it across the network, as the available reserves are being depleted.

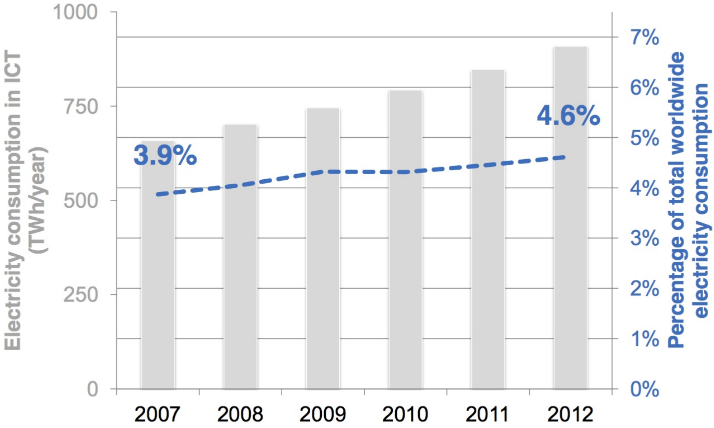
The tutorial will focus on both aspects, considering 5G applications and scenarios.

- The first half of the tutorial will discuss resource allocation methods for energy efficiency maximization.
- The second half of the tutorial will discuss energy harvesting and transfer methods.

Survey paper on energy management techniques in 5G

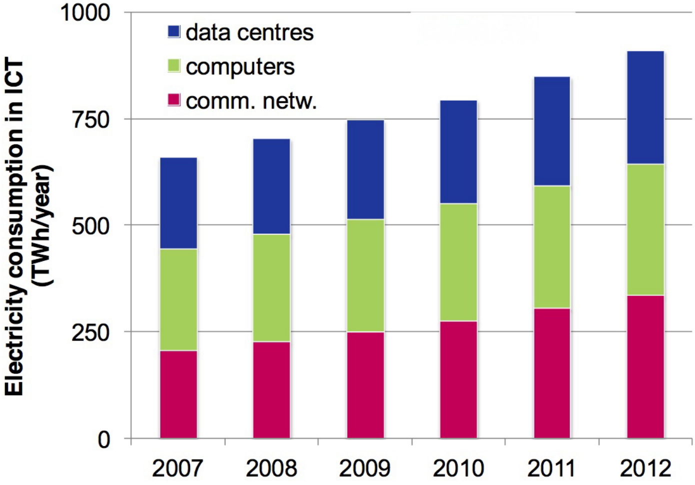
S. Buzzi, C.-L. I, T. E. Klein, H. V. Poor, C. Yang, A. Zappone
"A Survey of Energy-Efficient Techniques for 5G Networks and Challenges Ahead"
IEEE Journal on Selected Areas in Communications: Special Issue on Energy-efficient techniques for 5G, vol. 34, no. 4, April 2016.

ICT energy consumption[1]



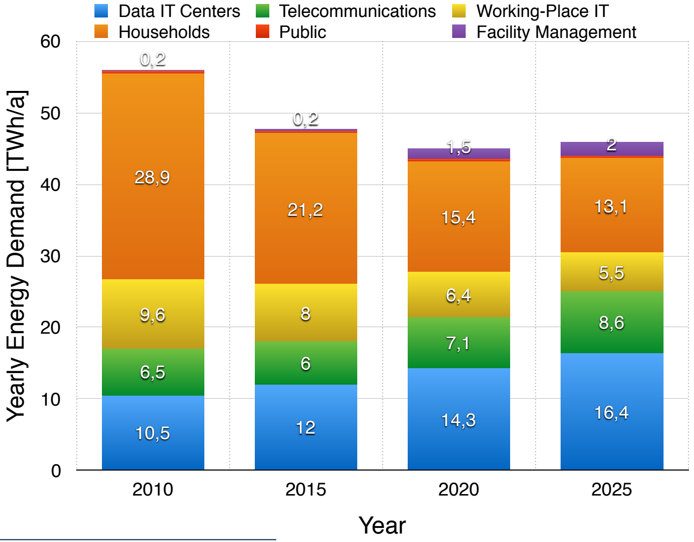
[1] TREND Final Workshop Brussels, October 24, 2013

Communication networks energy consumption[2]



[2] TREND Final Workshop Brussels, October 24, 2013

Energy demand prediction in Germany[3]



[3] 3rd study for ICT energy demand in Germany, 18. November 2015

And the numbers are rapidly increasing, especially with 5G networks!

- Exponential increase of devices (50 billion by 2020) and data traffic.[4] [5] [6]
- Battery lifetimes to increase by a factor 10 or more (especially for M2M communications)

"Energy efficiency is defined as the number of bits which can be transmitted per Joule of energy. 5G should support a 1000 times traffic increase in the next 10 years timeframe, with an energy consumption by the whole network of only half that is typically consumed by today's networks. This leads to the requirement of an energy efficiency increase of x2000 in the next 10 years timeframe."[7]

"5G will bring drastic energy efficiency improvement and develop energy harvesting everywhere. This energy chase will cover terminal devices, network elements, and the network as a whole including data centers."[8]

[4] Ericsson, "More than 50 billion connected devices", White Paper, February 2011

[5] Cisco, "Global Mobile Data Traffic Forecast Update," 2010–2015 White Paper, February 2011

[6] Nokia Siemens, "Networks 2011, 2020: Beyond 4G Radio Evolution for the Gigabit Experience", White Paper, February 2011

[7] Next Generation Mobile Network (NGMN) alliance 5G white paper," February 2015.

[8] "5G Public Private Partnership 5G manifesto", Mobile World Congress, March 2015.

Energy Efficiency Maximization

Tutorial/Survey on Resource Allocation for Energy Efficiency

A. Zappone and E. A. Jorswieck
"Energy Efficiency in Wireless Networks via Fractional Programming Theory"
Now Publisher, Foundations and Trends in Communications and Information Theory, vol. 11, no. 3-4, pp. 185-396, 2015.

Web Link: "`https://tu-dresden.de/die_tu_dresden/fakultaeten/fakultaet_elektrotechnik_und_informationstechnik/ifn/tnt/Now%20Tutorial%20EE`"

1. Tools for energy efficiency maximization.

2. Applications to 5G technologies.

Acknowledgements

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- Prof. M. Debbah, Huawei France, France.
- Prof. E. Björnson, Linköping University, Sweden.

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1. **Tools for energy efficiency maximization.**

The *energy efficiency* is defined as the system benefit-cost ratio in terms of amount of data reliably transmitted over the energy that is required to do so.

- When transmitting with power p for in the time slot T, the amount of transmitted information is proportional to $Tf(\gamma(p))$.
- The function $f(\cdot)$ is any measure of the amount of data that can be reliably sent to the destination per unit of time (i.e. the achievable rate, throughput, ...).

[9] G. Auer at al. "How much energy is needed to run a wireless network?" *IEEE Wireless Communications*, vol. 18, no. 5, pp. 40-49, 2011.

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- The corresponding energy consumption is $T(\mu p + P_c)$.
- $\mu \geq 1$ accounts for amplifier non-idealities and P_c for the power dissipated in all other hardware components (DA/AD converters, modulation filters, signal processing operation, ...)[9].

[9] G. Auer at al. "How much energy is needed to run a wireless network?" *IEEE Wireless Communications*, vol. 18, no. 5, pp. 40-49, 2011.

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Summing up, the energy efficiency is

$$\mathrm{EE} = \frac{f(\gamma(p))}{\mu p + P_c} \quad [\mathrm{bit/Joule}]$$

[9] G. Auer at al. "How much energy is needed to run a wireless network?" *IEEE Wireless Communications*, vol. 18, no. 5, pp. 40-49, 2011.

Typical shape of the EE

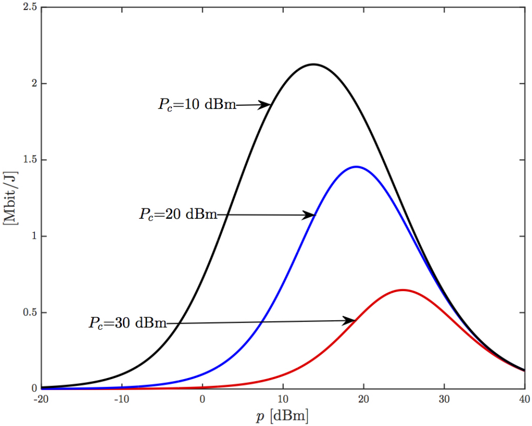


Figure : $\frac{W \log_2(1+\gamma)}{p+P_c}$

The EE is not concave and does not always increase with the transmit power

Requirements of a valid EE

$$\mathrm{EE} = \frac{f(\gamma(p))}{\mu p + P_c}$$

Not all functions f result in a physically meaningful EE. The EE should fulfill the following properties

- EE is measured in bit/Joule. So f should be measured in bit/s.
- $\mathrm{EE}(p) \geq 0$ for all $p \geq 0 \longleftrightarrow f(p) \geq 0$ for all $p \geq 0$
- $\mathrm{EE}(p = 0) = 0 \longleftrightarrow f(0) = 0$
- $\mathrm{EE}(p \to \infty) = 0.$ This means that $f(p)$ should grow slowlyer than a line.
- Canonical choices for $f(\gamma)$ are the achievable rate $W \log_2(1 + \gamma)$, with W the communication bandwidth, or the throughput $R(1 - e^{-\gamma})$, with R the communication rate. Other choices are also possible.

What if we have a network with K communication links? How should we combine the EEs of each link to define the EE of the network? Several choices are possible:

1. Global Energy Efficiency

$$\text{GEE} = \frac{\sum_{k=1}^{K} f(\gamma_k(\{p_k\}_{k=1}^{K}))}{\sum_{k=1}^{K} \mu_k p_k + P_{c,k}} \ .$$

It is the ratio between the global benefit and global cost of the network, but does not allow to tune the EE of the individual links.

2. Weighted arithmetic mean of the EEs

$$\text{Sum-EE} = \sum_{k=1}^{K} w_k \frac{f(\gamma_k(\{p_k\}_{k=1}^{K}))}{\mu_k p_k + P_{c,k}} \ .$$

It allows to tune the EE of the individual links by a suitable choice of the weights (very useful in heterogeneous networks).

Energy efficiency of a communication network

3. Weighted geometric mean of the EEs

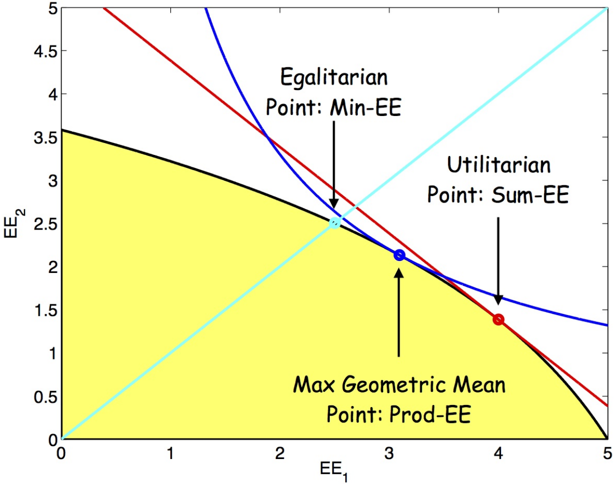
$$\text{Prod-EE} = \prod_{k=1}^{K} \left(\frac{f(\gamma_k(\{p_k\}_{k=1}^{K}))}{\mu_k p_k + P_{c,k}} \right)^{w_k} .$$

It ensures a more balanced and fair resource allocation, with no link experiencing a very low EE.

4. Weighted minimum EE

$$\text{Min-EE} = \min_{k} \left(w_k \frac{f(\gamma_k(\{p_k\}_{k=1}^{K}))}{\mu_k p_k + P_{c,k}} \right) .$$

It is a worst-case approach. No user experiencing a very low EE, but this may come at the expense of global performance.



- Varying the weights we can obtain different points on the Pareto boundary.
- In general, the GEE is inside the Pareto region, because it does not directly depend on the individual EEs, but has a clearer physical meaning from a global perspective.

Regardless of the definition, we have to maximize ratios
so we need fractional programming!

Questions?

Fractional programming in wireless networks

Communications Theory

Literature overview:

- Fractional programming in cognitive radio systems [1, 2, 3].
- Fractional programming in OFDMA systems [4, 5, 6].
- Fractional programming in MIMO systems [7, 8, 9].

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Fractional programming in wireless networks

- Fractional programming in large/Massive MIMO systems [10, 11, 12, 13].
- Fractional programming in relay-assisted systems [14, 15, 16].
- Fractional programming for secure wireless communications [17, 18, 19].

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- Fractional programming in CoMP systems [20, 21, 22, 23].
- Fractional programming for distributed resource allocation [24, 25, 13].

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Many more references can be found in [26].

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Definition: Fractional program

Let $f, g : \mathcal{P} \subseteq \mathbb{R}^n \to \mathbb{R}$, with $g(\boldsymbol{p}) > 0$, for all $\boldsymbol{p} \in \mathcal{P}$.

$$\max_{\boldsymbol{p}} \frac{f(\boldsymbol{p})}{g(\boldsymbol{p})} \quad \text{s.t.} \quad \boldsymbol{p} \in \mathcal{P}$$

- Maximization problems are easy to solve when the objective is concave. [10]
- First-order optimality (Karush-Kuhn-Tucker conditions) is necessary and sufficient for global optimality and well-known optimization algorithms exist.
- In general a fraction is not concave, not even assuming concave, convex, or linear f and g.
- Is there a wider class of functions that extends the properties of concave functions and that includes (at least some) fractional functions?

The answer is yes: *Pseudo-concave and quasi-concave functions*

[10] S. Boyd and L. Vandenberghe, "Convex Optimization," Cambridge University Press, 2004

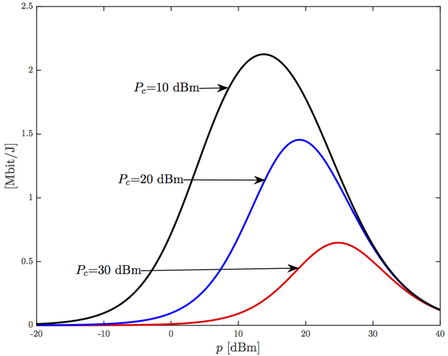
Quasi-concavity and Pseudo-concavity

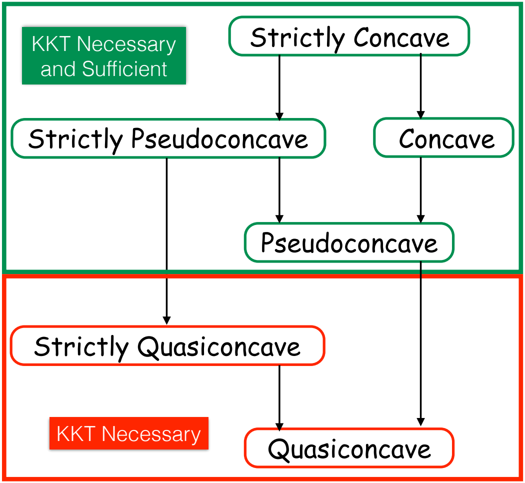
Let $\mathcal{C} \subseteq \mathbb{R}^n$ be a convex set. Then $f : \mathcal{C} \to \mathbb{R}$ is quasi-concave if, for all $\boldsymbol{p}_1, \boldsymbol{p}_2 \in \mathcal{C}$ and $\lambda \in [0;1]$:

$$f(\lambda \boldsymbol{p}_1 + (1-\lambda)\boldsymbol{p}_2) \leq \max\{f(\boldsymbol{p}_1), f(\boldsymbol{p}_2)\} \; .$$

Instead, f is pseudo-concave if and only if, for all $\boldsymbol{p}_1, \boldsymbol{p}_2 \in \mathcal{C}$, it is differentiable and:

$$f(\boldsymbol{p}_2) < f(\boldsymbol{p}_1) \Rightarrow \nabla(f(\boldsymbol{p}_1))^T(\boldsymbol{p}_2 - \boldsymbol{p}_1) < 0$$




KKT Necessary and Sufficient
Strictly Concave
Strictly Pseudoconcave
Concave
Pseudoconcave
Strictly Quasiconcave
KKT Necessary
Quasiconcave

When is a fractional function pseudo-concave or quasi-concave?

$$r(\boldsymbol{p}) = \frac{f(\boldsymbol{p})}{g(\boldsymbol{p})}$$

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Affine/Affine

$f(\boldsymbol{p})$, $g(\boldsymbol{p}) > 0$, affine. $r(\boldsymbol{p})$ is pseudo-concave and pseudo-convex (i.e. pseudo-linear)

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Concave/Affine

$f(\boldsymbol{p})$ differentiable and concave, $g(\boldsymbol{p}) > 0$ affine. $r(\boldsymbol{p})$ is pseudo-concave.

Generalized concavity of ratios

When is a fractional function pseudo-concave or quasi-concave?

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Affine/Affine

$f(\boldsymbol{p})$, $g(\boldsymbol{p}) > 0$, affine. $r(\boldsymbol{p})$ is pseudo-concave and pseudo-convex (i.e. pseudo-linear)

Concave/Affine

$f(\boldsymbol{p})$ differentiable and concave, $g(\boldsymbol{p}) > 0$ affine. $r(\boldsymbol{p})$ is pseudo-concave.

Concave/Convex

$f(\boldsymbol{p}) \geq 0$ differentiable and concave, $g(\boldsymbol{p}) > 0$ differentiable and convex. $r(\boldsymbol{p})$ is pseudo-concave. $r(\boldsymbol{p})$ is quasi-concave if the differentiability assumption is relaxed.

Pseudo-concave maximization

$f(\boldsymbol{p}) \geq 0$, $g(\boldsymbol{p}) > 0$, differentiable functions. f, concave, g, h_k convex for all $k = 1, \ldots, K$.

$$\max_{\boldsymbol{p}} \frac{f(\boldsymbol{p})}{g(\boldsymbol{p})}$$
$$\text{s.t. } h_k(\boldsymbol{p}) \leq 0 \ , \ \forall \ k = 1, \ldots, K$$

- The objective is pseudo-concave.
- First-order optimality implies global optimality.
- In principle we could solve (1) from first-order optimality conditions, i.e. (roughly speaking) by setting the gradient of the objective to zero.
- This naive approach might be numerically unstable due to the fractional form.

Fractional programming provides methods
to solve (1) by solving concave maximizations.

Consider the maximization problem, with $\lambda \geq 0$:

$$\max_{\boldsymbol{p}} \{ (f(\boldsymbol{p}) - \lambda g(\boldsymbol{p})) \ : \ h_k(\boldsymbol{p}) \leq 0 \ , \ \forall \ k = 1, \ldots, K \} \tag{1}$$

- We have a concave objective, since f is concave and g is convex.
- It would be nice to maximize f/g by solving concave problems like (1).

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- It would be nice to maximize f/g by solving concave problems like (1).

It turns out that we can!
We only have to find the right λ.

Dinkelbach's algorithm

$\epsilon > 0$; $n = 0$; $\lambda_n = 0$;
repeat

$$\boldsymbol{p}_n^* = \arg\max_{\boldsymbol{p}} \{f(\boldsymbol{p}) - \lambda_n g(\boldsymbol{p}) \; : \; h_k(\boldsymbol{p}) \leq 0 \, , \, \forall \, k = 1, \ldots, K\};$$

$$F(\lambda_n) = f(\boldsymbol{p}_n^*) - \lambda_n g(\boldsymbol{p}_n^*);$$

$$\lambda_{n+1} = \frac{f(\boldsymbol{p}_n^*)}{g(\boldsymbol{p}_n^*)};$$

$$n = n + 1;$$

until $F(\lambda_n) < \epsilon$

Insights into Dinkelbach's algorithm

- Upon convergence we obtain a $\boldsymbol{p}^*$ which is the global solution of the fractional problem.
- The update rule for λ follows Newton's method and so we have a super-linear convergence rate
- Ideally, upon convergence we have $F(\lambda) = 0$.

Indeed, we have the following result:

Theorem [27]

Consider the function

$$F(\lambda) = \max_{\boldsymbol{p}} \{ (f(\boldsymbol{p}) - \lambda g(\boldsymbol{p})) \ : \ h_k(\boldsymbol{p}) \leq 0 \ , \ \forall \, k = 1, \ldots, K \} \tag{2}$$

There exists a unique, positive λ^* such that $F(\lambda^*) = 0$ and an optimal solution of (2) with $\lambda = \lambda^*$ solves the CFP.

References

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Dinkelbach's algorithm is not limited to concave/convex maximizations.

- Can be extended to maximize the minimum of a family of ratios, i.e.

$$\max_{\boldsymbol{p}} \min_{1 \leq i \leq I} \frac{f_i(\boldsymbol{p})}{g_i(\boldsymbol{p})}$$

- Can work also when f is not concave and/or g is not convex, but in this case $f(\boldsymbol{p}) - \lambda g(\boldsymbol{p})$ is not a concave function and we should *globally* solve a non-concave problem in each iteration.

Instead, the case of a sum of ratios or product of ratios are in general more involved.

- Even assuming each ratio has the concave/convex structure, it is not possible to convert the problem into concave maximizations without loss of optimality.
- In these cases we have to trade-off optimality with complexity.

Extensions of Dinkelbach's algorithm

Dinkelbach's algorithm is not limited to concave/convex maximizations.

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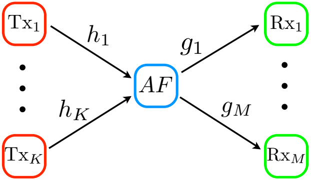
Take-Home Points:

- Single-ratios and max-min problems with the concave/convex structure can be globally solved with limited complexity
- Other cases in general require exponential complexity to be globally solved

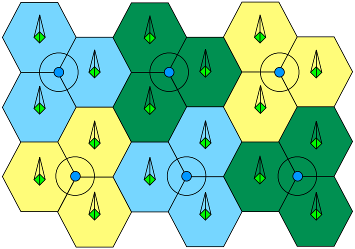
- **2. Applications to 5G technologies**

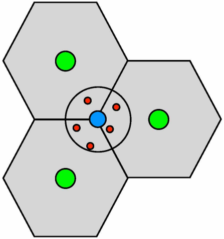
We consider a heterogeneous interference network with K transmitters, M receivers, and (possibly) one AF relay. Multiple antennas and subcarriers can be employed.



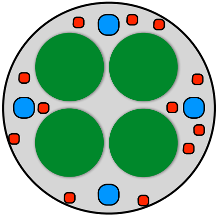
- Shared relay in heterogeneous multi-cell systems.
- Infrastructure sharing in heterogeneous multi-cell systems.
- Heterogeneous network with small-cells.
- We can also model hardware-impaired systems and systems with imperfect channel estimation (typical of massive MIMO).
- Full-duplex systems.
- Peer-to-peer networks (if $K = M$), such as device-to-device communications.

- Shared Relay (standardized in LTE-A) and infrastructure sharing. 3-cell clusters with half-duplex relays.





- Heterogeneous small-cells. Relays are deployed to serve cell-edge users.



- **2.1 Network Energy Efficiency Maximization in 5G Networks [13]**

References

[13] A. Zappone, L. Sanguinetti, G. Bacci, E. A. Jorswieck, and M. Debbah, "Energy-efficient power control: A look at 5G wireless technologies," *IEEE Transactions on Signal Processing*, vol. 64, no. 7, pp. 1668–1683, April 2016

Let us consider the maximization of the GEE:

GEE Maximization

$$\max_{\boldsymbol{p}} \frac{\sum_{k=1}^{K} \log_2(1+\gamma_k(\boldsymbol{p}))}{\sum_{k=1}^{K} p_k + P_{c,k}} \tag{3a}$$

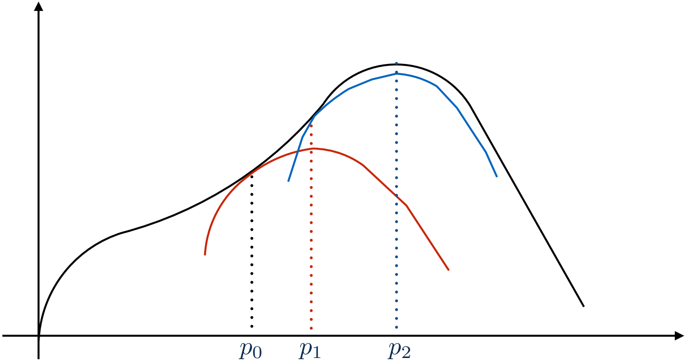
$$\text{s.t. } 0 \leq p_k \leq P_{max,k}\ ,\ \log_2(1+\gamma_k(\boldsymbol{p})) \geq R_{min,k}\ ,\ \forall\ k = 1,\ldots,K \tag{3b}$$

- The SINR is expressed as $\gamma_k = \dfrac{\alpha_k p_k}{\sigma_k^2 + \sum_{j \neq k} p_j \omega_{j,k} + \phi_k p_k}$.
- A non-zero ϕ_k allows modeling hardware impairments, relay systems, imperfect channel estimation, frequency-selective channels, inter-symbol interference.
- α_k, ϕ_k, $\omega_{j,k}$ are specialized depending on the particular system under analysis. Setting $\phi_k = 0$ yields the usual SINR expression of cellular networks.
- Necessary and sufficient feasibility conditions can be derived [13] (not related to fractional programming)

The numerator of (3a) is not concave in all transmit powers.
We can not directly use fractional programming to maximize the GEE.
This is a general problem in interference-limited networks!

How do we use fractional programming
if the numerator of our objective is not concave?

We solve a sequence of easier fractional problems
where each problem can be solved by fractional programming.



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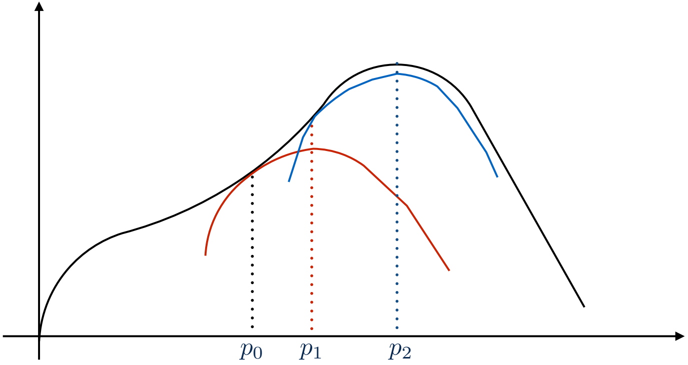

This approach can lower the complexity, but what about its property?
Does it converge? If yes, does the limit point enjoy any optimality property?

Sequential Programming

Let $\mathcal{P}$ be a maximization problem with objective $r_0(\boldsymbol{p})$ and constraints $r_i(\boldsymbol{p})$, $i = 1, \ldots, I$. Then, consider a sequence of Problems $\{\mathcal{P}_j\}_j$, with objective $r_{0,j}(\boldsymbol{p})$, constraints $r_{i,j}(\boldsymbol{p})$, and solutions $\{\boldsymbol{p}_j^*\}_j$. Assume that for any i, j it holds:

1. $r_{i,j}(\boldsymbol{p}) \leq r_i(\boldsymbol{p})$, for all $\boldsymbol{p}$, $i = 0, \ldots, I$, and j.
2. $r_{i,j}(\boldsymbol{p}_{j-1}^*) = r_i(\boldsymbol{p}_{j-1}^*)$, $i = 0, \ldots, I$, and j.
3. $\nabla r_{i,j}(\boldsymbol{p}_{j-1}^*) = \nabla r_i(\boldsymbol{p}_{j-1}^*)$, $i = 0, \ldots, I$, and j.

This approach provides two main optimality properties:

- The solutions of the problems $\{\mathcal{P}_j\}_j$ monotonically increase the original objective, i.e. $r_0(\boldsymbol{p}_j^*) \geq r_0(\boldsymbol{p}_{j-1}^*)$ for all j.
- So, we have convergence in the objective (assuming the objective is upper-bounded on the feasible set).
- The algorithm ends when $|r_0(\boldsymbol{p}_j^*) - r_0(\boldsymbol{p}_{j-1}^*)| \leq \epsilon$.
- Upon convergence, the objective value corresponds to a first-order optimal solution for the original problem $\mathcal{P}$.

Clearly, the approach is useful provided the Problems $\{\mathcal{P}_j\}_j$ can be easily solved (e.g. because they are pseudo-concave fractional problems)

Sequential fractional programming

To handle the GEE maximization problem we can use the approximation:

$$\log_2(1+\gamma) \geq a \log_2 \gamma + b$$

wherein $a = \frac{\gamma_0}{1+\gamma_0}$ and $b = \log_2(1+\gamma_0) - \frac{\gamma_0}{1+\gamma_0} \log_2 \gamma_0$.

It can be shown that all three properties are fulfilled
but does the GEE numerator become concave?

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It can be shown that all three properties are fulfilled
but does the GEE numerator become concave?

$$\begin{aligned}\log_2(1+\gamma_k) &\geq [a_k\log_2(\gamma_k) + b_k] \\ &= \left[a_k\log_2(\alpha_k p_k) - a_k\log_2\left(\sigma_k^2 + \phi_k p_k + \sum_{j\neq k}\omega_{j,k}p_j\right) + b_k\right]\end{aligned}$$

Are we done?

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$$\begin{aligned} \log_2(1+\gamma_k) &\geq \left[a_k \log_2(\gamma_k) + b_k\right] \\ &= \left[a_k \log_2(\alpha_k p_k) - a_k \log_2\left(\sigma_k^2 + \phi_k p_k + \sum_{j \neq k} \omega_{j,k} p_j\right) + b_k\right] \end{aligned}$$

Are we done?

Not yet, but after the variable change $\boldsymbol{p} = 2^{\boldsymbol{q}}$ we obtain:

$$\widetilde{R}_k(\boldsymbol{q}) = \left[a_k \log_2(\alpha_k) + a_k q_k - a_k \log_2\left(\sigma_k^2 + \phi_k 2^{q_k} + \sum_{j \neq k} \omega_{j,k} 2^{q_j}\right) + b_k\right]$$

Concave function in all variables!

So, we can implement the sequential method by solving problems of the form:

$$\max_{\boldsymbol{q}} \frac{\sum_{k=1}^{K} \widetilde{R}_k(\boldsymbol{q})}{\sum_{k=1}^{K} 2^{q_k} + P_{c,k}} \tag{4a}$$

$$\text{s.t. } 2^{q_k} \leq P_{max,k} \ , \ \widetilde{R}_k(\boldsymbol{q}) \geq R_{min,k} \ , \ \forall \ k = 1, \ldots, K \tag{4b}$$

Sequential fractional programming

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GEE maximization

Select a feasible $\boldsymbol{p}$;
Set $\gamma_k = \gamma_k(\boldsymbol{p})$ and compute $\tilde{R}_k$ for all k;
repeat
  Solve (4) and call the solution $\boldsymbol{q}$;
  $\boldsymbol{p} = 2^{\boldsymbol{q}}$; $\gamma_k = \gamma_k(\boldsymbol{p})$ and update $\tilde{R}_k$ for all k;
until convergence

Sequential fractional programming

Communications Theory

So, we can implement the sequential method by solving problems of the form:

$$\max_{\boldsymbol{q}} \frac{\sum_{k=1}^{K} \widetilde{R}_k(\boldsymbol{q})}{\sum_{k=1}^{K} 2^{q_k} + P_{c,k}} \quad (4a)$$

$$\text{s.t. } 2^{q_k} \leq P_{max,k} \ , \ \widetilde{R}_k(\boldsymbol{q}) \geq R_{min,k} \ , \ \forall \, k = 1, \ldots, K \quad (4b)$$

GEE maximization

Select a feasible $\boldsymbol{p}$;
Set $\gamma_k = \gamma_k(\boldsymbol{p})$ and compute $\tilde{R}_k$ for all k;
repeat
  Solve (4) and call the solution $\boldsymbol{q}$;
  $\boldsymbol{p} = 2^{\boldsymbol{q}}$; $\gamma_k = \gamma_k(\boldsymbol{p})$ and update $\tilde{R}_k$ for all k;
until convergence

Result

After each iteration the GEE value increases, and upon convergence we obtain a first-order optimal value for the original problem.

The same approach can be applied in other scenarios.

Min-EE Maximization

$$\max_{\boldsymbol{p}} \min_{1\leq k\leq K} \frac{\log_2(1+\gamma_k(\boldsymbol{p}))}{p_k + P_{c,k}} \tag{5a}$$

$$\text{s.t. } 0 \leq p_k \leq P_{max,k} \ , \ \forall\, k = 1, \ldots, K \tag{5b}$$

$$\log_2(1+\gamma_k(\boldsymbol{p})) \geq R_{min,k} \ , \ \forall\, k = 1, \ldots, K \tag{5c}$$

Multi-carrier scenarios

$$\max_{\boldsymbol{p}} \frac{\sum_{k=1}^{K}\sum_{n=1}^{N} \log_2(1+\gamma_{k,n}(\boldsymbol{p}))}{\sum_{k=1}^{K}\sum_{n=1}^{N} p_{k,n} + P_c} \tag{6a}$$

$$\text{s.t. } \sum_{n=1}^{N} p_{k,n} \leq P_{max,k} \ , \ \forall\, k = 1, \ldots, K \tag{6b}$$

$$\sum_{n=1}^{N} \log_2(1+\gamma_{k,n}(\boldsymbol{p})) \geq R_{min,k} \ , \ \forall\, k = 1, \ldots, K \tag{6c}$$

The k-th user's rate can be upper bounded as

$$\log_2\left(1+\frac{\alpha_k p_k}{\sigma_k^2+\sum_{j\neq k} p_j\omega_{j,k}+\phi_k p_k}\right)\leq \log_2\left(1+\frac{\alpha_k}{\phi_k}\right)=R_{max,k} \qquad (7)$$

The rate constraint parameters $R_{min,k}$ have been set as a percentage of $R_{max,k}$.
$P_{max,1}=,\ldots,=P_{max,K}=\overline{P}$

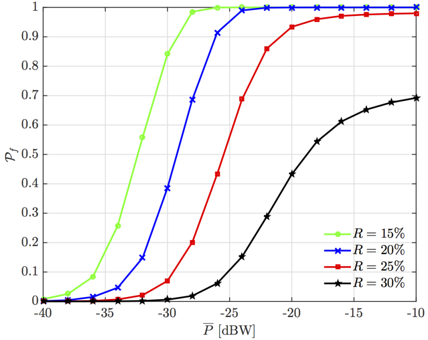

Figure : $K=5; M=50$. Probability of feasibility $\mathcal{P}_f$ versus $\overline{P}$ with minimum per user-rate constraints: (a) $R=15\%$; (b) $R=20\%$; (c) $R=25\%$; (d) $R=30\%$.

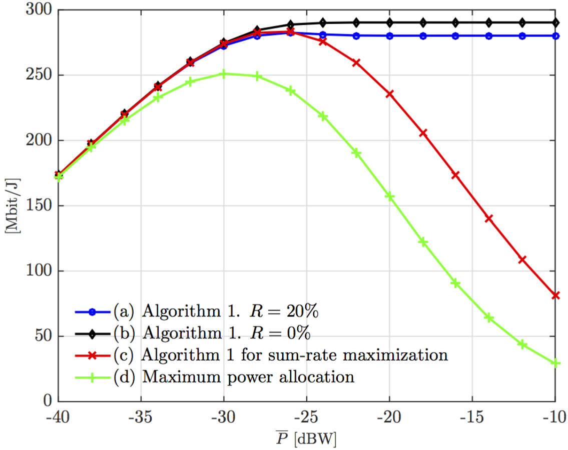

Figure : $K = 5; M = 50$. GEE versus P_{max} for: (a) GEE maximization with $R = 20\%$; (b) GEE maximization with $R = 0\%$; (c) sum-rate maximization; (d) Maximum transmit power allocation.

2.2 Self-organizing Energy Efficiency Maximization in 5G Networks [13]

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So far we have considered centralized schemes where all users cooperate to maximize a common objective function (e.g. GEE).

What about distributed scenarios where the users are competitive?

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What about distributed scenarios where the users are competitive?

The problem is mathematically formulated as

$$\begin{aligned} &\max_{p_k} \ \frac{\log_2(1+\gamma_k(p_k,\boldsymbol{p}_{-k}))}{p_k+P_{c,k}} && \forall k=1,\ldots,K \\ &\text{s.t.} \quad p_k \in [0;P_{max,k}]\ ,\ \log_2(1+\gamma_k(p_k,\boldsymbol{p}_{-k})) \geq R_{min,k} && \forall k=1,\ldots,K \end{aligned} \tag{8}$$

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This problem is challenging because:

- The K individual problems are coupled through $\gamma_k(p_k,\boldsymbol{p}_{-k})$.
- $\gamma_k = \dfrac{\alpha_k p_k}{\phi_k p_k + \omega_k}$, with $\omega_k = \sigma_k^2 + \sum_{j\neq k}\omega_{j,k}p_j$.
- Does (8) admit one or more equilibria? Does (8) converge?

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- Does (8) admit one or more equilibria? Does (8) converge?

But first of all: why do we need to consider distributed scenarios?
Because of the low feedback and complexity, which enable self-organizing networks.

Solving the individual problems

$$\max_{p_k} \frac{\log_2\left(1+\dfrac{\alpha_k p_k}{\phi_k p_k+\omega_k}\right)}{p_k+P_{c,k}}$$
$$\text{s.t.} \quad p_k \in [0; P_{max,k}]$$
$$\log_2(1+\gamma_k) \geq R_{min,k}$$

- The numerator is concave in p_k and the denominator is affine in p_k.
- We have a pseudo-concave objective.

The global solution can be obtained from first-order optimality conditions (i.e. setting the derivative of the objective to zero and accounting for the constraints)

$$p_k^* = \min(P_{max,k}, \max(\bar{p}_k, P_{min,k})) \ ,$$

with $\bar{p}_k$ the unique stationary point of the energy efficiency and

$$P_{min,k} = (2^{R_{min,k}}-1)\frac{\sigma_k^2+\sum_{j\neq k}\omega_j p_j}{\alpha_k-\phi_k(2^{R_{min,k}}-1)}$$

The problem is unfeasible if $P_{max,k} < P_{min,k}$

Convergence and distributed implementation

- If each individual problem is feasible, a unique equilibrium point exists.
- The unique equilibrium can be reached by iteratively solving the individual problems until convergence.

The proof exploits game-theoretic tools, but fractional programming plays a role.
The result holds upon showing the quasi-concavity of the individual energy efficiencies.

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The proof exploits game-theoretic tools, but fractional programming plays a role.
The result holds upon showing the quasi-concavity of the individual energy efficiencies.

Distributed Power Control

For all $k = 1, \ldots, K$, initialize p_k to feasible values.
repeat
  for Each k **do**
    Compute $p_k^* = \min(P_{max,k}, \max(\bar{p}_k, P_{min,k}))$.
  end for
until Convergence

But how do we implement this in a distributed way?

Distributed implementation

The k-th user's SINR is given by

$$\gamma_k = \frac{\alpha_k p_k}{\phi_k p_k + \omega_k}$$

The problem is ω_k because:

$$\omega_k = \underbrace{\sigma_k^2}_{\text{Equivalent noise}} + \underbrace{\sum_{j \neq k} p_j \omega_{j,k}}_{\text{Multi-user interference}}$$

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BRD algorithm

For all $k = 1, \ldots, K$, initialize p_k to a feasible value;
repeat
  for Each player k **do**
    At receiver k, measure the SINR γ_k;
    Compute $\omega_k = \frac{\alpha_k p_k}{\gamma_k} - \phi_k p_k$.
    Compute $p_k^* = \min(P_{max,k}, \max(\bar{p}_k, P_{min,k}))$ and send it to transmitter k.
  end for
until Convergence is reached

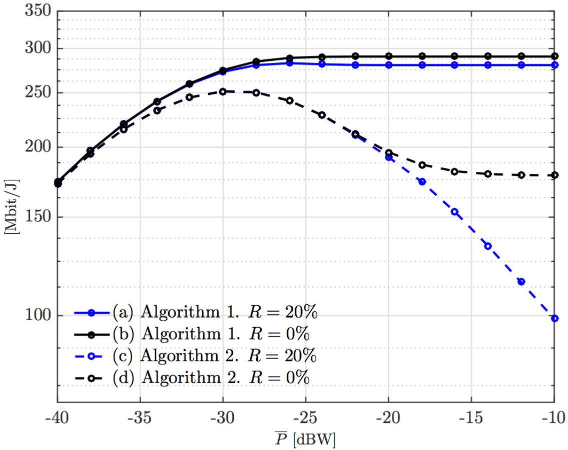

Figure : $K = 5; M = 50$. GEE versus P_{max} for: (a) Centralized GEE maximization with $R = 20\%$; (b) Centralized GEE maximization with $R = 0\%$; (c) Distributed GEE maximization with $R = 20\%$; (d) Distributed GEE maximization with $R = 0\%$.

Iterations for convergence

	$\overline{P} = -34$	$\overline{P} = -26$	$\overline{P} = -18$	$\overline{P} = -10$
Centralized. $R = 0\%$	3.69	6.30	6.49	6.51
Centralized. $R = 20\%$	3.67	6.68	6.76	6.77
Distributed. $R = 0\%$	1.01	1.42	3.66	4.50
Distributed. $R = 20\%$	1.01	1.42	3.67	6.71

What have we seen?

- Sequential fractional programming framework for centralized power control for 5G (and in general for interference networks).
- Self-organizing power control for 5G (and in general for interference networks).
- The framework applies to (massive) MIMO, multi/small-cells, heterogeneous networks, device-to-device, multi-carrier, full-duplex systems.
- We have seen only power control, but the framework can be extended to optimize other resources, e.g. beamforming vectors/matrices, receive filters, subcarrier scheduling, also assuming statistical CSI [9, 22, 15, 16]
- Many more details and examples to be found in [26]

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Ongoing work

- Global optimality? Numerical evidence suggests yes [28].
- Energy efficiency in device-to-device communications [29, 30] and in systems using physical layer security [31, 32].
- Impact of overhead transmissions (feedback, backhaul) on energy efficiency?.
- Combining different 5G techniques (e.g. resource allocation plus energy harvesting).

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Thank you for listening

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